

* The Absolute Value Function | دالة القيمة المطلقة

$$|x| = \begin{cases} x & , x \geq 0 \\ -x & , x < 0 \end{cases} \quad \text{تعريف القيمة المطلقة}$$

* Properties of absolute value :-

- ① $|-a| = |a|$
- ② $|ab| = |a| |b|$
- ③ $|\frac{a}{b}| = \frac{|a|}{|b|}$, $b \neq 0$
- ④ $|a+b| \leq |a| + |b|$

EX :- Find the solution of :-

① $|x-4| < 2$?

Sol: $x-4 < 2 \Rightarrow x < 6$

$-(x-4) < 2 \xrightarrow{\times -1} x-4 > -2 \Rightarrow x > 2$

\therefore the sol. $\{2 < x < 6\}$

② $|2x-5| > 1$?

Sol: $2x-5 > 1 \Rightarrow 2x > 6 \Rightarrow x > 3$

$-(2x-5) > 1 \xrightarrow{\times -1} 2x-5 < -1 \Rightarrow 2x < 4$

$x < 2$

\therefore The sol. $\{3 < x < 2\}$

③ $-6 < 2x+6 < 4$?

sol: $-6 < 2x+6 \Rightarrow -12 < 2x \Rightarrow x > -6$

$2x+6 < 4 \Rightarrow 2x < -2 \Rightarrow x < -1$

\therefore The sol. $\{-6 < x < -1\}$.

④ $x^2 - 5x + 6 \leq 0$? على هذا المثال يكون

sol: $(x-2)(x-3) \leq 0$ غير دقيق

~~$x-2 \leq 0 \Rightarrow x \leq 2$ and $x-3 \leq 0 \Rightarrow x \leq 3$~~

~~$x-2 \geq 0 \Rightarrow x \geq 2$, $x-3 \geq 0 \Rightarrow x \geq 3$~~

~~\therefore The sol. $\{2 \leq x \leq 3\}$~~

الاجابة الصحيحة هي $\{2, 3\}$

* Inverse function معكوس الدالة

الدالة g تكون معكوس الدالة f بشرط
 $g(f(x)) = x$

Ex: - Find the inverse function with prove of

① $y = 3x$?

④ $y = \frac{x}{\sqrt{1-x^2}}$ Hint

Sol: $y = 3x \Rightarrow x = \frac{y}{3}$

$g(f(x)) = x \Rightarrow g(\frac{y}{3}) = x \Rightarrow g(\frac{3x}{3}) = x$

② $y = \frac{2}{x-1}$?

Sol: $y(x-1) = 2 \Rightarrow yx = 2+y \Rightarrow x = \frac{2+y}{y}$

$g(f(x)) = x \Rightarrow g(\frac{2+y}{y}) = g(\frac{2 + \frac{2}{x-1}}{\frac{2}{x-1}}) \Rightarrow$

$g(\frac{\frac{2x-1+2}{x-1}}{\frac{2}{x-1}}) = g(\frac{2x}{2}) = x$

③ $y = \frac{3}{x^2+2}$?

Sol: $yx^2 + 2y = 3 \Rightarrow x^2 = \frac{3-2y}{y} \Rightarrow x = \sqrt{\frac{3-2y}{y}}$

$g(\sqrt{\frac{3-2y}{y}}) = g(\sqrt{\frac{3 - \frac{6}{x^2+2}}{\frac{3}{x^2+2}}}) = g(\sqrt{\frac{\frac{3x^2+6-6}{x^2+2}}{\frac{3}{x^2+2}}})$

$g(\sqrt{\frac{3x^2}{3}}) = x$